SH Wave Propagation in Semiconductor/Piezoelectric Structures

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Abstract-SH wave propagation in a piezoelectric half-space covered by a semiconductor film with initial stress is investigated analytically. The equilibrium equations for semiconductors with initial stress are presented. The dispersion relation of SH waves is obtained for different thickness of the film and wavenumber.

Ⅰ. INTRODUCTION

Acoustic wave propagating in a piezoelectric crystal is usually accompanied by an electric field. When the crystal is also semiconducting, the electric field produces currents and space charge resulting in dispersion and acoustic loss. The interaction between a traveling acoustic wave and mobile charges in piezoelectric semiconductors is called an acoustoelectric effect. It was also found that an acoustic wave traveling in a piezoelectric semiconductor can be amplified by application of a DC electric field[1]. This phenomenon is called acoustoelectric amplification of acoustic waves. The acoustoelectric effect and amplification of acoustic waves can also be achieved through composite structures of piezoelectric dielectrics and nonpiezoeletric semiconductors, which have led to the development of acoustoelectric devices. Usually, these kind devices are made of layered structure where a thin or thick layer is deposited on elastic or piezoelectric substrate. Because of the mismatch of material properties, the layers are most often subjected to high internal residual stress in piezoelectric devices. In the meantime, the layered piezoelectric structure is usually pre-stressed during the manufacture process to avoid brittle fracture. As a result, the effect of the initial stress on the propagation of SH surface wave has remarkable importance for design of devices. These devices must operate under various environmental conditions. For example, they can be used at room temperature or in very hot environments. Because of the mismatch of material properties, the layers are most often subjected to high internal residual stress in piezoelectric devices. In the meantime, the layered piezoelectric structure is usually pre-stressed during the manufacture process to avoid brittle fracture. As a result, the effect of the initial stress on the propagation of SH wave has remarkable importance for design of the sensors. Numerous investigations have been undertaken for the characteristic analysis of SH-SAW or Love waves in layered piezoelectric structures by researchers in various disciplines because of its important applications in SAW devices. Dowaikh[2] examined the SH waves in a pre-stressed layered half space for an incompressible elastic material. Liu[3] et al. investigated the effect of initial stress on the propagation behavior of Love waves in a layered piezoelectric structure. Due to multi-field coupling and anisotropy, device modeling by these theories presents complicated mathematical problems. Wauer[4] derived the exact solutions for one-dimensional problem of thickness vibration of plates, but this analysis is possible only in rare cases. Yang[5] had investigated acoustoelectric amplification of piezoelectric surface waves, obtained the propagation of anti-plane surface wave solution.

In this paper, SH wave propagation in a piezoelectric half-space covered by a semiconductor film with initial stress is investigated analytically. The equilibrium equations for semiconductors with initial stress are presented. The dispersion relation of SH waves is obtained for different thickness of the film and wavenumber. The effect of the initial stresses on the phase velocity is discussed in detail for piezoelectric ceramics PZT-5H and semiconductor silicon.

Ⅱ. FORMULATION OF THE PROBLEM

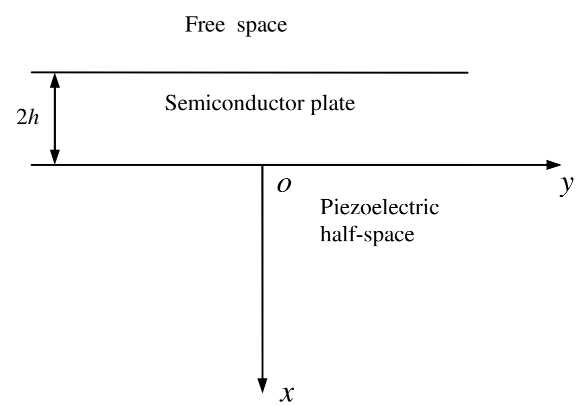

Fig.1 A piezoelectric half-space covered with a semiconductor layer

Consider a transversely isotropic semi-infinite piezoelectric substrate covered with a semiconductor material layer as illustrated in Fig.1. The piezoelectricity is polarized in the z direction. The equilibrium equations of elasticity without body forces and the Gauss's law of electrostatics without free charge are given as follows

$$\sigma_{ji,j} = \rho \ddot{u}_i \, , D_{i,i} = 0 \, , i, \, j = 1,2,3 \, , \tag{1}$$

where σ_{ij} is the stress tensor, D_i is the electric displacement, and ρ is the mass density of the piezoelectric material.

On the assumption that the SH waves propagate in the y direction, the total out-of-plane displacement and the electric potential are expressed as

$$u = v = 0, \;\; w = w(x, y, t) \;, \;\; \phi = \phi(x, y, t) \;, \tag{2}$$

where u, v, w are the elastic displacement components, respectively, and ϕ is the electric potential. The extended strain-displacement relations are

$$\gamma_{xz} = w_{,x}, \quad \gamma_{yz} = w_{,y}, \tag{3}$$

$$E_x = -\phi_{,x}, \quad E_y = -\phi_{,y}, \tag{4}$$

where E is the electric field. The constitutive relations can be written as

$$\begin{aligned} \tau_{xz} &= c_{44}\frac{\partial w}{\partial x} + e_{15}\frac{\partial \phi}{\partial x}, \quad D_x = e_{15}\frac{\partial w}{\partial x} - \varepsilon_{11}\frac{\partial \phi}{\partial x}, \\ \tau_{yz} &= c_{44}\frac{\partial w}{\partial y} + e_{15}\frac{\partial \phi}{\partial y}, \quad D_y = e_{15}\frac{\partial w}{\partial y} - \varepsilon_{11}\frac{\partial \phi}{\partial y}. \end{aligned} \tag{5}$$

For a semiconductor layer, the electric current can be expressed as

$$\begin{aligned} J_y &= q\bar{n}\mu_{11}E_y + qn\mu_{11}\bar{E}_y - qd_{11}n_{,y} \\ &= -q\bar{n}\mu_{11}\phi_{,y} + qn\mu_{11}\bar{E}_y - qd_{11}n_{,y}. \end{aligned} \tag{6}$$

where q is the carrier charge which may be the electronic charge or its negative, $\bar{n}$ is the steady-state carrier density which produces electrical neutrality, and μ_{ij} is the carrier mobility. $\bar{E}_i$ is the uniform DC electric field. When an acoustic wave propagates through the material, perturbations of the electric field, the carrier density and the current are denoted by E_i, n and J_i.

Ⅲ. SOLUTION OF THE PROBLEM

3.1. Solution for a semi-infinite piezoelectric substrate

Substituting equations (3-5) into equation (1), we can obtain

$$\begin{aligned} &c_{44}^m \nabla^2 w^m + e_{15}^m \nabla^2 \phi^m = \rho^m \frac{\partial^2 w^m}{\partial t^2}, \\ &e_{15}^m \nabla^2 w^m - \varepsilon_{11}^m \nabla^2 \phi^m = 0, \end{aligned} \tag{7}$$

where superscript m indicates the quantities of the substrate. By assume

$$\psi^m = \phi^m - \frac{e_{15}^m}{\varepsilon_{11}^m} w^m, \tag{8}$$

the equation (7) can be rewritten as

$$c_{44}^{m*}\nabla^2 w^m = \rho^m \frac{\partial^2 w^m}{\partial t^2}, \ \nabla^2 \psi^m = 0, \tag{9}$$

where $c_{44}^{m*} = c_{44}^m + \frac{e_{15}^m}{\varepsilon_{11}^m}$.

The solutions of the equation (9) must satisfy the following radiation conditions

$$x \to +\infty, \quad w^m, \psi^m \to 0. \tag{10}$$

The solutions of the equations (9) are assumed as follows

$$\begin{aligned} w^m &= A\exp(-kb_A x)e^{i(ky-\omega t)}, \\ \psi^m &= B\exp(-kx)e^{i(ky-\omega t)}, \end{aligned} \tag{11}$$

where A and B are unknown constants to be determined. Substituting equation (11) into (9), we can obtain

$$b_A^2 = 1 - \rho c^2 / c_{44}^{m*} = 1 - c^2 / c_{sh}^{m\,2}, \tag{12}$$

where c is the phase velocity of the waves and $c = \omega / k$. c_{sh}^m is the velocity of the shear horizontal wave and $c_{sh}^m = \sqrt{c_{44}^{m*}/\rho^m}$. Substituting equation (12) into (8), we can obtain

$$\phi^m = (Be^{-kx} + A\frac{e_{15}^m}{\varepsilon_{11}^m}e^{-kb_A x})e^{i(ky-\omega t)}, \tag{13}$$

$$\tau_{xz}^m = -k(Ac_{44}^{m*}b_A e^{-kb_A x} + Be_{15}^m e^{-kx})e^{i(ky-\omega t)}, \tag{14}$$

$$D_x^m = B\varepsilon_{11}^m k e^{-kx} e^{i(ky-\omega t)}. \tag{15}$$

3.2. Solution of the semiconductor with initial stress

$$\sigma_{ij,j} + (u_{i,k}\sigma_{kj}^0)_{,j} = \rho\ddot{u}_i, \tag{16}$$

$$D_{i,i} = qn, \tag{17}$$

$$q\dot{n} + J_{i,i} = 0. \tag{18}$$

Substituting equation (1~6) into (16~18)， we can obtain

$$c_{44}\nabla^2 w + e_{15}\nabla^2\phi + \frac{\partial^2 w}{\partial y^2}\sigma_y^0 = \rho\frac{\partial^2 w}{\partial t^2}, \tag{19}$$

$$e_{15}\nabla^2 w - \varepsilon_{11}\nabla^2\phi = qn, \tag{20}$$

$$\dot{n} - \bar{n}\mu_{11}\phi_{,yy} + \mu_{11}\bar{E}_y n_{,y} - d_{11}n_{,yy} = 0. \tag{21}$$

The solutions of the equations (19)-(21) are assumed as follows

$$w = Ae^{i(ky-\omega t)}, \tag{22}$$

$$\phi = Ce^{i(ky-\omega t)}, \tag{23}$$

$$n = Ne^{i(ky-\omega t)}, \tag{24}$$

Silicon is a cubic crystal with m3m symmetry which does not have piezoelectric coupling.

Substituting equations (22)-(24) into (5)-(6), we can obtain

$$\tau_{yz} = c_{44}\frac{\partial w}{\partial y} = Ac_{44}ike^{i(ky-\omega t)}, \tag{25}$$

$$D_y = -\varepsilon_{11}\frac{\partial \phi}{\partial y} = -C\varepsilon_{11}ike^{i(ky-\omega t)}, \tag{26}$$

$$J_y = (-Cq\bar{n}\mu_{11}ik + q\mu_{11}\bar{E}_y N - qd_{11}ikN)e^{i(ky-\omega t)}. \tag{27}$$

3.3. Solution of the vacuum

The electric potential in the vacuum must satisfy Laplace equation

$$\begin{aligned} &\nabla^2\phi^0 = 0, \qquad x < 0. \\ &\phi^0 \to 0, \qquad x \to -\infty. \end{aligned} \tag{28}$$

The solutions of the equations (28) are assumed as follows

$$\phi^0 = Ce^{kx}e^{i(ky-\omega t)}, \tag{29}$$

$$D_x^0 = -C\varepsilon_0 k e^{kx} e^{i(ky-\omega t)}. \tag{30}$$

Ⅳ. THE PHASE OF VELOCITY EQUATIONS

Consider the semiconductor plate of the thickness $2h$, equations (22-24) can rewritten as follows

$$\frac{\partial \tau_{yz}}{\partial y} + \frac{1}{2h}[\tau_{xz}(x=0) - \tau_{xz}(x=-2h)] + \frac{\partial^2 w}{\partial y^2}\sigma_y^0 = \rho\ddot{w}, \tag{31}$$

$$D_{y,y} + \frac{1}{2h}[D_x(x=0) - D_x(x=-2h)] = qn, \tag{32}$$

$$q\dot{n} + J_{y,y} + \frac{1}{2h}[J_x(x=0) - J_x(x=-2h)] = 0. \tag{33}$$

4.1 Boundary of the open condition at the free surface

According to boundary condition and continuity condition

$$B + A\frac{e_{15}^m}{\varepsilon_{11}^m} = C, \tag{34}$$

$$A(c_{44}k^2+\frac{1}{2h}kb_A c_{44}^{m*}+k^2\sigma_y^0-\rho\omega^2)+\frac{1}{2h}Bke_{15}^m=0, \quad (35)$$

$$C(k^2\varepsilon_{11}+\frac{1}{2h}k\varepsilon_0 e^{-2kh})+\frac{1}{2h}Bk\varepsilon_{11}^m-qN=0, \quad (36)$$

$$Cq\bar{n}\mu_{11}k^2+N[-q\omega i+q\mu_{11}\bar{E}_y ki+qd_{11}k^2]=0, \quad (37)$$

In order to obtain the nontrivial solutions of the above-mentioned unknown constants A,B,C,N , the determinant of the coefficient matrix of liner algebraic equations (34)-(37) have to be equal to zero. So we can obtain the dispersion relation of SH waves as follows

$$b^2\frac{c_{44}}{c_{44}^{m*}}2kh-b_A+\frac{e_{15}^{m2}}{\varepsilon_{11}^m c_{44}^{m*}} =\frac{\frac{e_{15}^{m2}}{\varepsilon_{11}^m c_{44}^{m*}}}{1+\frac{\varepsilon_0}{\varepsilon_{11}^m}e^{-2kh}+\frac{\varepsilon_{11}}{\varepsilon_{11}^m}2kh+\frac{q\bar{n}\mu_{11}2h}{\varepsilon_{11}^m[d_{11}k+i(\mu_{11}\bar{E}_y-c)]}}. \quad (38)$$

where $b^2=\frac{\rho c^2-\sigma_y^0}{c_{44}}-1.$

4.2 Boundary of the short condition at the free surface

The electrically short condition can lead to

$$Ck^2\varepsilon_{11}+\frac{1}{2h}Bk\varepsilon_{11}^m-qN=0. \quad (39)$$

We can obtain the dispersion relation of SH waves as

$$b^2\frac{c_{44}}{c_{44}^{m*}}2kh-b_A+\frac{e_{15}^{m2}}{\varepsilon_{11}^m c_{44}^{m*}} =\frac{\frac{e_{15}^{m2}}{\varepsilon_{11}^m c_{44}^{m*}}}{1+\frac{\varepsilon_{11}}{\varepsilon_{11}^m}2kh+\frac{2q\bar{n}\mu_{11}h}{\varepsilon_{11}^m[d_{11}k+i(\mu_{11}\bar{E}_y-c)]}}. \quad (40)$$

V. NUMERICAL RESULTS

The mobility of electron and holes of silicon at $300\mathrm{K}^0$ [6] is

$$\mu_n=1500,\quad \mu_p=480\mathrm{cm^2/V-sec}. \quad (41)$$

The diffusion constants can be determined from the Einstein relation[6]

$$D=\frac{kT}{q_e}\mu. \quad (42)$$

At room temperature[6] $\frac{kT}{q_e}=0.026\mathrm{V}$, where $q_e=1.602\times10^{-19}$. γ is a dimensionless number given by

$$\gamma=\mu_{11}\bar{E}_1/c_{B-G}. \quad (43)$$

5.1 Solution of the open condition at the free surface

When $h=0$, the semiconductor film does not exist, equation (38) reduces to

Table 1 Constants of the Materials

	$\rho(\mathrm{kg/m^3})$	$c_{44}(10^{10}\,\mathrm{N/m^2})$	$\varepsilon_{11}(10^{-12}\,\mathrm{farads/m})$	$e_{15}(\mathrm{C/m^2})$
PZT-5H	7500	2.3	$1700\varepsilon_0$	17
Si	2332	7.596	$11.8\varepsilon_0$	0

$$c^2=c_{sh}^{m2}(1-\frac{(\frac{e_{15}^{m2}}{\varepsilon_{11}^m c_{44}^{m*}})^2}{1+\varepsilon_{11}/\varepsilon_0})=c_{B-G}^2. \quad (44)$$

When $c_{(0)}=\frac{\omega}{k}=\mu_{11}\bar{E}_1$, the acoustic wave speed is equal to that of the carrier drift. As the lowest (zero) order of approximation, we neglect the semiconduction and denote the zero-order solution by $c_{(0)}$. For the next order we substitute $c_{(0)}$ into the right-hand side of (38) and obtain the following equation for $c_{(1)}$

$$b^2\frac{c_{44}}{c_{44}^{m*}}2kh-b_A+\frac{e_{15}^{m2}}{\varepsilon_{11}^m c_{44}^{m*}} =\frac{\frac{e_{15}^{m2}}{\varepsilon_{11}^m c_{44}^{m*}}}{1+\frac{\varepsilon_0}{\varepsilon_{11}^m}e^{-2kh}+\frac{\varepsilon_{11}}{\varepsilon_{11}^m}2kh+\frac{q\bar{n}\mu_{11}2h}{\varepsilon_{11}^m[d_{11}k+i(\mu_{11}\bar{E}_y-c_{(0)})]}}. \quad (45)$$

5.2. Solution of the short condition at the free surface

When $h=0$, we can obtain the velocity of the B-G wave

$$c^2=c_{sh}^{m2}[1-(\frac{e_{15}^{m2}}{\varepsilon_{11}^m c_{44}^{m*}})^2]=c_{B-G}^2. \quad (46)$$

When $c_{(0)}=\frac{\omega}{k}=\mu_{11}\bar{E}_1$, , substituting (46) into (40),we can obtain the following equation for $c_{(1)}$

$$b^2\frac{c_{44}}{c_{44}^{m*}}2kh-b_A+\frac{e_{15}^{m2}}{\varepsilon_{11}^m c_{44}^{m*}} =\frac{\frac{e_{15}^{m2}}{\varepsilon_{11}^m c_{44}^{m*}}}{1+\frac{\varepsilon_{11}}{\varepsilon_{11}^m}2kh+\frac{q\bar{n}\mu_{11}2h}{\varepsilon_{11}^m[d_{11}k+i(\mu_{11}\bar{E}_y-c_{(0)})]}}. \quad (47)$$

The calculation results of the phase velocity versus initial stress are illustrated as Figs. 2-7.

Figs.2-3 shows that the dispersion relations of the zero-order for the electrically open and short cases with different tensile initial stress .it can be seen that the phase velocity decreases with the increases of the wave number under the same initial stress.

Figs.4-5 shows that the dispersion relations of the zero-order for the electrically open and short cases with different compressive initial stresses .We can find that the phase velocity increases with the increases of the wave number under the same initial stress.

Figs.6 shows that the dispersion relations of the first-order with $\gamma = 2$ for the electrically short case with different compressive initials stress.

Figs.7 shows that the dispersion relations of the first-order with $\gamma = 2$ and the zero-order for the electrically short cases with tensile initial stress

Ⅵ. CONCLUSION

SH wave propagation in a piezoelectric half-space covered by a semiconductor film with initial stress is investigated analytically. The equilibrium equations for semiconductors with initial stress are presented. The dispersion relation of SH waves is obtained for different initial stresses in the film and different wavenumber. The effect of the initial stress on the phase velocity is discussed in detail. We can find that the initial stress has remarkable effect on the phase velocity of the SH waves.

Acknowledgment

This work is supported by the Key Industrial Project of Ningbo Bureau of Science and Technology (No.2005B100015) and a grant from the Science and Technology Division, Zhejiang Province, China, under Key Technological Initiative Program (Project No. 2006C14021).

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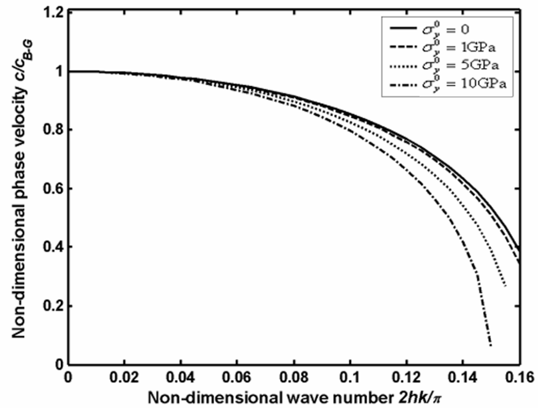

Fig.2 Dispersion relations of zero-order for the electrically short case with different tensile initial stress

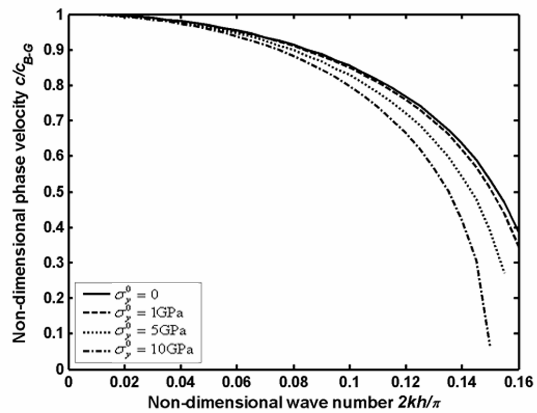

Fig.3 Dispersion relations of zero-order for the electrically open case with different tensile initial stress

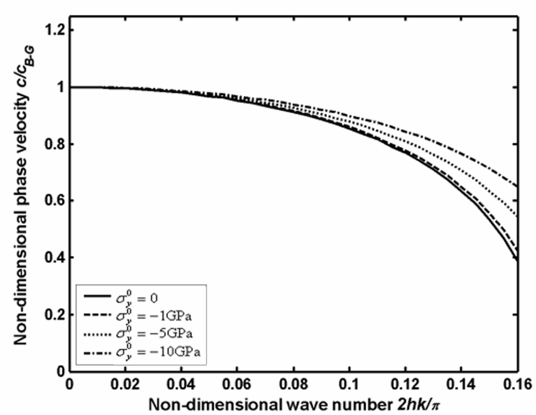

Fig.4 Dispersion relations of zero-order for the electrically short case with different compressive initials stress

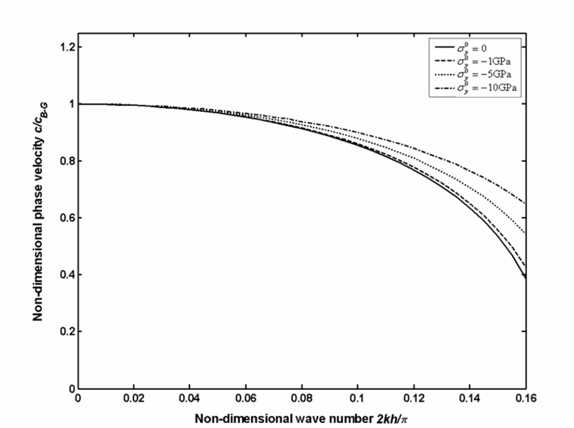

Fig.5 Dispersion relations of zero-order for the electrically open case with different compressive initial stress

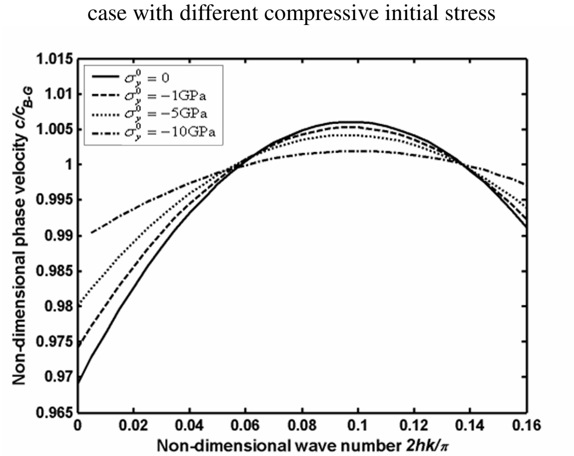

Fig.6 Dispersion relations of the first-order with $\gamma = 2$ for the electrically short case under different compressive initial stress

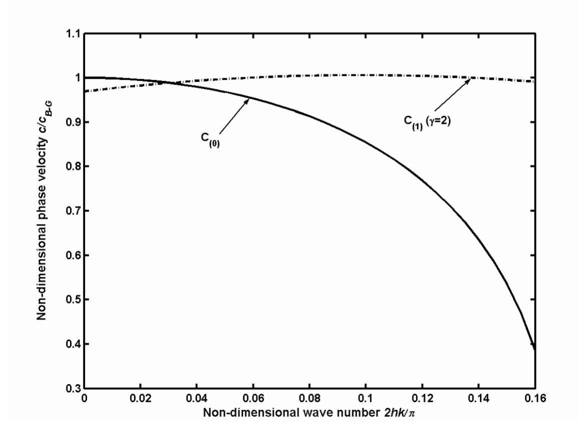

Fig.7 Dispersion relation of the first-order with $\gamma = 2$ and the zero-order for the electrically short case with tensile initial stress